METHODS OF NONDESTRUCTIVE THERMOPHYSICAL CHECKING OF ANISOTROPIC BODIES

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Formulas are obtained to determine the components of the thermal-diffusivity tensor of anisotropic bodies by using the integral temperature and heat flux characteristics.

Methods to determine the thermophysical coefficients, which are based on surface thermal probing of fabricated articles without destroying them, are of indubitable interest for the possibility of performing thermophysical measurements directly on bodies of arbitrary geometry [1-4].

As a rule, known methods of thermophysical testing assume the use of minimum thermal information (for example, a temperature measurement at one point of the surface at a definite time [1]) in order not to complicate execution of the experiment. However, this results in awkward analytical expressions in computations in nondestructive testing methods, when only one side of the body surface is accessible to observation.

In our opinion, reduction of additional information about the body surface temperature in tests and its representation in the form of integral characteristics [5] permit raising the confidence in the measurements and simplification of the computational formulas by averaging the results obtained.

New methods of nondestructive thermophysical testing of articles on the basis of a general utilization of integral characteristics are elucidated below.

§1. Absolute Method of Determining the Thermophysical Coefficients of a Semibounded Body

Let us first examine the problem of determining the thermophysical characteristics of a semibounded orthotropic body whose thermal-diffusivity tensor components are considered constant. Let us assume that the principal axes of the tensor $\begin{pmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{pmatrix}$ are known. Let us direct the ox, oy, and oz coordinate axes along the principal directions by directing ox and oz along the surface and oy into the body by considering the heating to be in the y = 0 plane. The heat-conduction equation in this case will have the form

$$\frac{\partial U}{\partial t} - a_1 \frac{\partial^2 U}{\partial x^2} - a_2 \frac{\partial^2 U}{\partial y^2} - a_3 \frac{\partial^2 U}{\partial z^2} = 0,\ t > 0, \quad -\infty < x,\ z < \infty,\ y > 0, \tag{1}$$

where U = U(t, x, y, z) is the body temperature.

Let us consider the initial temperature constant and equal to zero, the heat flux density q going into the body through the surface y = 0 to be a symmetric function relative to the ox axis and independent of the coordinate z, i.e.,

$$U|_{t=0} = 0,\ -\lambda_2 \frac{\partial U}{\partial y}\bigg|_{y=0} = q(t,\ x), \tag{2}$$

where $\lambda_1 = a_1 c$, $\lambda_2 = a_2 c$, $\lambda_3 = a_3 c$ are the thermal-conductivity coefficients along the ox, oy,

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and oz axes, respectively, and c is the volume specific heat. Then $\partial^2U/\partial z^2 = 0$ and $U(t, x, y, z) \equiv U(t, x, y)$.

Let us call integrals of the form

$$\tilde{U}^*(p, s, y) = \int_0^\infty \int_0^\infty U(t, x, y) \exp(-pt) \cos sx dt dx,$$
$$\tilde{q}^*(p, s) = \int_0^\infty \int_0^\infty q(t, x) \exp(-pt) \cos sx dt dx \tag{3}$$

the integral characteristics of the body temperature and the heat flux.

Values of these characteristics on the body surface y = 0 are interrelated by the relationship

$$\tilde{U}^*(p, s, 0) = \frac{\tilde{q}^*(p, s)}{\theta_2 \sqrt{p + a_1 s^2}}, \quad \theta_2 = \frac{\lambda_2}{\sqrt{a_2}}. \tag{4}$$

An expression for the coefficient of thermal diffusivity a_1 along the body surface follows from (4):

$$a_1 = \frac{p_2 \left(\frac{\tilde{q}^*(p_1, s)}{\tilde{U}^*(p_1, s, 0)} \right)^2 - p_1 \left(\frac{\tilde{q}^*(p_2, s)}{\tilde{U}^*(p_2, s, 0)} \right)^2}{s^2 \left[\left(\frac{\tilde{q}^*(p_2, s)}{\tilde{U}^*(p_2, s, 0)} \right)^2 - \left(\frac{\tilde{q}^*(p_1, s)}{\tilde{U}^*(p_1, s, 0)} \right)^2 \right]}, \tag{5}$$

as does the thermal activity $\theta_2 = \lambda_2/\sqrt{a_2}$ in the bulk of the body:

$$\theta_2 = \frac{\tilde{q}^*(p_1, s)}{\tilde{U}^*(p_1, s, 0) \sqrt{p_1 + a_1 s^2}} = \frac{\tilde{q}^*(p_2, s)}{\tilde{U}^*(p_2, s, 0) \sqrt{p_2 + a_1 s^2}}. \tag{6}$$

These equalities are valid for all p, s if a_1 and θ_2 are constants, i.e., (5) and (6) can be considered the necessary condition for a_1 and θ_2 to be constant.

The component a_3 is determined analogously if the heating is along the ox axis.

§2. Relative Method of Determining the Thermophysical Coefficient of a Semibounded Body

Under the assumptions and notation from Sec. 1, a standard body with the known thermophysical characteristics a_s and λ_s in thermal contact with the article under investigation (Fig. 1) is used in addition.†

Let a plane heater, located between the bodies, liberate a total heat flux of density $q_t(t, x)$. Let us assume the heater thickness to be negligibly small and its specific heat to be much less than the specific heats of the specimen and the standard, i.e.,

$$q_t(t, x) = q(t, x) - q_s(t, x), \; q(t, x) > 0, \; q_s(t, x) < 0,$$
$$U_s(t, x, 0) = U(t, x, 0).$$

The equality of the fluxes and temperatures of the specimen and standard surfaces yields equality of their integral characteristics

$$\tilde{q}_t^*(p, s) = \tilde{q}^*(p, s) - \tilde{q}_s^*(p, s),$$
$$\tilde{U}_s^*(p, s, 0) = \tilde{U}^*(p, s, 0). \tag{7}$$

†Because $\partial^2 u/\partial z^2 = 0$, the coefficients a_3, λ_3 will not be encountered later and it is impossible to determine them for such a heating scheme.

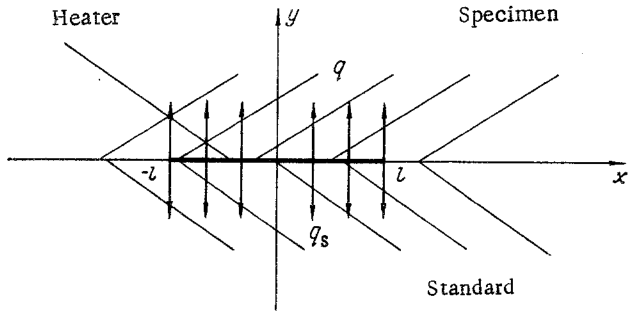

Fig. 1. Heating diagram for the system under investigation.

$$\frac{-\tilde{q}_s^*(p, s)}{\theta_s\sqrt{p+a_s s^2}} = \frac{\tilde{q}^*(p, s)}{\theta_2\sqrt{p+a_1 s^2}}, \quad \theta_s = \frac{\lambda_s}{\sqrt{a_s}}. \tag{8}$$

Solving (7) and (8) jointly, we obtain

$$\tilde{U}^*(p, s, 0) = \frac{\tilde{q}_s^*(p, s)}{\theta_s\sqrt{p+a_s s^2}+\theta_2\sqrt{p+a_1 s^2}}. \tag{9}$$

Let us use two equations to find a_1 and θ_2:

$$\tilde{U}^*(p_i, s, 0) = \frac{\tilde{q}_t^*(p_i, s)}{\theta_s\sqrt{p_i+a_s s^2}+\theta_2\sqrt{p_i+a_1 s^2}}, \quad i=1, 2.$$

from which

$$a_1 = \frac{p_2\left(\frac{A_1}{A_2}\right)^2 - p_1}{s^2\left[1-\left(\frac{A_1}{A_2}\right)^2\right]}, \tag{10}$$

$$\theta_2 = \frac{A_1}{\sqrt{p_1+a_1 s^2}} = \frac{A_2}{\sqrt{p_2+a_1 s^2}}, \tag{11}$$

where

$$A_i = \frac{\tilde{q}_t^*(p_i, s)}{\tilde{U}^*(p_i, s, 0)} - \theta_s\sqrt{p_i+a_s s^2}, \quad i=1, 2.$$

§3. Determination of the Thermophysical Characteristics of an Anisotropic Bounded Body

Let us consider the general case of an anisotropic body with surface Γ occupying a finite domain V of three-dimensional space. Let us assume V to be given in a fixed Cartesian coordinate system. To simplify the writing let x_1, x_2, and x_3 denote the coordinate axes.

Let us assume that the temperature g(t, x), $x = (x_1, x_2, x_3)$ of the surface Γ and the heat flux q(t, x) through this surface are measured simultaneously in the same thermal process. The initial temperature $\varphi(x)$ and the intensity of the internal heat sources f(t, x), $x \in V$ are also known. Let us consider the volume specific heat c and the heat-conduction tensor to be constant. Then the process of heat propagation in the body is described by the problem

$$\frac{\partial U(t, x)}{\partial t} - \sum_{j,k=1}^{3} a_{jk}\frac{\partial^2 U(t, x)}{\partial x_j \partial x_k} = \frac{1}{c} f(t, x), \quad a_{jk} = a_{kj}, \tag{12}$$

$$t > 0, \ x \in V, \ x = (x_1, x_2, x_3),$$

$$U(0, x) = \varphi(x), \ x \in V, \tag{13}$$

$$U(t, x) = g(t, x), \; t > 0, \; x \in \Gamma, \tag{14}$$

$$\sum_{j,k=1}^{3} a_{jk} \frac{\partial U(t, x)}{\partial x_k} \cos(n, x_j) = -\frac{1}{c} q(t, x), \quad t > 0, \; x \in \Gamma, \tag{15}$$

where $\bar{n} = \{\cos(n, x_1), \cos(n, x_2), \cos(n, x_3)\}$ is the unit vector of the external normal n to the surface Γ erected at the point x.

It is clear that it is impossible to consider the temperature g(t, x) and the heat flux q(t, x) independent in the problem (12)-(15), since it is sufficient to give one of the conditions, (14) or (15), in addition to the initial condition (13) for unique solvability of the "direct" problem.

The relationship between the surface temperature g(t, x) and the flux q(t, x) penetrating through this surface, as well as the thermal-diffusivity tensor, the initial function φ(x), and the source function f(t, x), is set up by using integral characteristics of a special kind. This relationship is the following.

If the matrix $(a_{jk})_1^3$ is a constant, symmetric, and positive definite and g(t, x), q(t, x), (x∈Γ), f(t, x), φ(x), (x∈V), t > 0 are bounded, then for all real p_j, j = 1, 2, 3, which do not vanish simultaneously, the equality

$$P(p, a_{jk}) \equiv \sum_{j,k=1}^{3} a_{jk} p_k \int_{\Gamma} \exp(-px)\, g^*(p_4, x) \cos(n, x_j)\, d\Gamma + \frac{1}{c} \int_{\Gamma} \exp(-px)\, q^*(p,x)\, d\Gamma + \int_{V} \exp(-px) \times \left(\frac{1}{c} f^*(p, x) + \varphi(x)\right) dx = 0, \quad px = \sum_{i=1}^{3} p_i x_i, \quad p_4 = \sum_{j,k=1}^{3} a_{jk} p_j p_k, \tag{16}$$

holds, where the asterisk denotes the Laplace transform of the appropriate function, for instance,

$$g^*(p_4, x) = \int_0^\infty \exp(-p_4 t)\, g(t, x)\, dt, \quad x = (x_1, x_2, x_3).$$

The equality (16) can be considered as the necessary condition which the matrix $(a_{jk})_1^3$ of the coordinates of the thermal-diffusivity tensor must satisfy if it is known a priori that the tensor belongs to a definite class: it is symmetric, i.e., the Onsager relationships are satisfied, $a_{jk} = a_{kj}$, and the matrix $(a_{jk})_1^3$ is positive definite, i.e., $\sum_{j,k=1}^{3} a_{jk} p_j p_k > 0$ for all real p_j such that $\sum_{j=1}^{3} p_j^2 > 0$. The condition of positive definiteness is perfectly natural here since it indicates that the heat-conduction equation (12) is parabolic.

The equality (16) can be used to obtain an algorithm to determine the coefficients a_{jk}. Let us introduce the functional

$$J(a_{jk}) = \int_{R^3} \rho(p) P^2(p, a_{jk})\, dp, \quad p = (p_1, p_2, p_3), \quad R^3 = \left\{\sum_{i=1}^{3} p_j^2 < \infty\right\},$$

where ρ(p) is an arbitrary positive finite function such that $\rho(p) \le |p|^\alpha$, α > 0, in the neighborhood of zero.

If (16) is satisfied, then $J(a_{jk})$ reaches its minimum value and hence

$$\frac{\partial J(a_{jk})}{\partial a_{jk}} = 0, \quad j, k = 1, 2, 3, \quad a_{jk} = a_{kj}. \tag{17}$$

The solution of the nonlinear system (17) should be sought so that the matrix $(a_{jk})_1^3$ would be positive definite.

§4. Example of an Approximate Calculation of the Integral Characteristics $\tilde{u}(p, s, 0)$ and $\tilde{q}_t^*(p, s)$ from (3)

Let us consider the problem of determining the integral characteristics of the surface temperature and the heat flux density in application to the problem in Sec. 2. Let us examine one important case when the heat flux supplied to the body through a surface strip of width $2l$ is constant within it:

$$q_t(t, x) = \begin{cases} q_t = \text{const}, & |x| \leqslant l, \; q_c \geqslant 0, \\ 0, & |x| > l. \end{cases}$$

In this case the integral characteristic of the heat flux density is written analytically:

$$\tilde{q}_t^*(p, s) = \frac{q_t \sin sl}{ps}, \tag{18}$$

and then (9) becomes

$$\tilde{U}^*(p, s, 0) = \frac{q_t \sin sl}{ps\left(\theta_s\sqrt{p + a_s s^2} + \theta_2\sqrt{p + a_1 s^2}\right)}. \tag{19}$$

For y = 0 the integral $\tilde{U}^*(p, s, 0)$ from (3) can be represented as

$$\tilde{U}^*(p, s, 0) = \int_0^\infty U^*(p, x, 0)\cos sx\,dx, \tag{20}$$

where

$$U^*(p, x, 0) = \int_0^\infty U(t, x, 0)\exp(-pt)\,dt \tag{21}$$

is the Laplace transform of the function U(t, x, 0).

Then using the inversion formula of the Fourier integral transform, we find the integral (21) from (19) and (20):

$$U^*(p, x, 0) = \frac{2q_t}{\pi p}\int_0^\infty \frac{\sin rl \cos rx}{\theta_s\sqrt{p + a_s r^2} + \theta_2\sqrt{p + a_1 r^2}}\,\frac{dr}{r}. \tag{22}$$

Furthermore, the value of $\tilde{U}^*(p, s, 0)$ is calculated approximately.

The integral (21) was initially evaluated for a number of values of x

$$U^*(p, x, 0) \approx \frac{1}{p}\sum_{k=1}^{n} b_k U\left(\frac{t_k}{p}, x, 0\right).$$

The coefficients were selected according to a table [6]. It turns out [5] that for n = 5 for all x_i, $0 \leq x_i < \infty$, in the ranges

$$0.1 \leqslant \frac{a_1}{a_s} \leqslant 5; \; 0.1 \leqslant \frac{\theta_2}{\theta_s} \leqslant 5; \; 1 \leqslant \frac{pl^2}{a_s} \leqslant 10,$$

the error in evaluating the integral did not exceed 0.02%.

Then we find the integral (20) approximately

TABLE 1

$sl=\frac{\pi}{2}$; $0,1\leqslant\frac{a_1}{a_s}\leqslant 5$; $0,1\leqslant\frac{\theta_2}{\theta_s}\leqslant 5$; $1\leqslant\frac{pl^2}{a_s}\leqslant 10$				
$\frac{x_i}{l}$	0	0,5	1	1,5
$\frac{c_i}{l}$	0,5756	0,6973	—1,3050	—0,0869

$$\tilde{U}^*(p,\ s,\ 0)\approx\frac{1}{s}\sum_{i=1}^{m}c_iU^*\left(p,\ \frac{x_i}{s},\ 0\right). \tag{23}$$

Substituting (19) and (22) into (23), we obtain

$$\frac{\sin sl}{\theta_s\sqrt{p+a_ss^2}+\theta_2\sqrt{p+a_1s^2}}\approx\frac{2}{\pi}\sum_{i=1}^{m}c_i\int_0^\infty\frac{\sin rl\cos r\frac{x_i}{s}}{\theta_s\sqrt{p+a_sr^2}+\theta_2\sqrt{p+a_1r^2}}\frac{dr}{r}. \tag{24}$$

The parameter s and the coefficients c_i are found on an ODRA-1204 computer by minimizing the difference between the left and right sides of (24) in the selected range of a_1/a_s, θ_2/θ_s, and pl^2/a_s.

The results of calculations for m = 4, reduced to dimensionless form, are presented in Table 1.

The error we obtained in calculating a_1 and θ_2 by means of (10) and (11), using the tabulated quadrature coefficients b_k [6] and the quadrature coefficients c_i, will not exceed 1%.

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